Pair production by a circularly polarized electromagnetic wave in a plasma

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(Received 24 July 2003; revised manuscript received 3 October 2003; published 30 March 2004)

We present the results of the calculation of the electron-positron pair creation probability by a circularly polarized electromagnetic wave during its propagation in underdense collisionless plasmas. The dependence of the probability on the frequency and the amplitude of the pulse is studied in detail.

I. INTRODUCTION

Quantum electrodynamics (QED) predicts the possibility of electron-positron pair production in a vacuum by strong electric field [1]. This nonlinear effect has attracted great attention due to the fact that it lays outside the scope of perturbation theory and sheds a light on the nonlinear quantum electrodynamics properties of the vacuum [2]. As was shown in Refs. [1,3] a planar electromagnetic wave of arbitrary intensity and spectral composition does not produce the electron-positron pairs in a vacuum because it has both the electromagnetic field invariants $J_1 = (\mathbf{B}^2 - \mathbf{E}^2)/2$ and $J_2 = (\mathbf{E} \cdot \mathbf{B})/2$ equal to zero [4]. Due to this reason the pair creation was first considered for a static electric field, which evidently has nonvanishing invariant J_1, then its theoretical description was extended on the time-varying electric-type electromagnetic field [5]. In this case the electromagnetic field invariants J_1 and J_2 obey conditions

$$J_1 = \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} = \tfrac{1}{2} (\mathbf{B}^2 - \mathbf{E}^2) < 0,$$

$$J_2 = \tfrac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} = \tfrac{1}{2} (\mathbf{E} \cdot \mathbf{B}) = 0, \tag{1}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor and $\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$, $\varepsilon^{\mu\nu\rho\sigma}$ is the totally antisymmetric tensor. In particular, the pair creation in a vacuum in a spatially homogeneous electric field

$$\mathbf{E}(t) = F\phi(t)\mathbf{e}_z, \qquad \mathbf{B}(t) = 0, \tag{2}$$

where $\phi(t) = \cos t$ and $\mathbf{e}_z$ is a unit vector in the z direction, was considered in Refs. [5–13].

The process (called the Schwinger pair production mechanism) is of fundamental importance in QED and in quantum field theory in general. However, previous estimates [7,14,15] showed that it hardly could be observed in the experiments using by that time available optical lasers whose power was far from the limit able to provide an intensity of the order of 10^{29} W/cm^2, which corresponds to the Schwinger critical electric field 1.32×10^{16} V/cm. As closer the electric field approaches the Schwinger limit as the sub-barrier tunneling results in more efficient electron-positron pair production. Therefore, the results of studies presented in

Refs. [1,5–11] for a long time were generally believed to be of purely theoretical interest in QED.

Recently, fortunately, the situation has drastically changed, because the power of optical and infrared lasers rose by many orders of magnitude [16]. This opened several ways to approach critical QED intensity. One way was demonstrated in the SLAC experiments [17], where 46.6 GeV electron beam interacted with counterpropagating laser pulse, whose intensity was 0.5×10^{18} W/cm^2. The incident radiation in its turn interacted with the similar laser pulse and several electron-positron pairs were detected. Then, the projects to build free-electron lasers at TESLA electron-positron collider in DESY and the corresponding facilities at SLAC, in which coherent laser beams with the photon energies of the order of several keV are supposed to be produced, are being designed [18] (see also Refs. [13,19,20]). Recently, an approach to the light intensification towards the Schwinger limit by using nonlinear interaction between the electromagnetic and the Langmuir wave in plasmas has been formulated in Ref. [21]. In this scheme the QED critical field can be achieved with present day laser systems. Hence the theory of the Schwinger effect must be considered in more detail in view of the new experimental possibilities.

However, here arises a question, whether the model of pair creation by the time-varying electrical field (2) in a vacuum resembles well enough the experimental situation. Such a field can be generated in the antinodes of the standing light wave, which is realized in a superposition of two counterpropagating laser beams. In this case, the region where electric field is much larger than the magnetic field is relatively small and the results obtained in Refs. [5–13,22] describe just a small portion of the pairs that could be created by the electromagnetic wave. In the rest part, the pair creation will be affected by the presence of the time-varying magnetic field. This in its turn should lead to very complicated formulas for pair production probability.

To elucidate the role of the magnetic field component on the electron-positron creation we consider below the planar circularly polarized electromagnetic wave propagating in the underdense collisionless plasma. Even in a very low density plasma the first electromagnetic invariant J_1 calculated for planar electromagnetic wave is not equal to zero. That is why in a plasma the electron-positron pairs can be created by the planar wave. In this case the usage of the Lorentz transformation into the reference frame moving with the wave group velocity leads to the electromagnetic wave field transformation to the purely electric field, which rotates with constant

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frequency, and with no magnetic field. Although this ansatz reduces the problem under consideration to the situation when the pairs are created by the time-varying electric field, actually, the wave magnetic field effects are incorporated rigorously into our model. We notice that similar approach has been earlier used in Ref. [27]. In the present paper we use the method of "imaginary time," simple and powerful mathematical technique for analytical continuation of the classical equations of motion postulated to apply inside the barrier, through the barrier where classical motion is not allowed to join up with the equations of motion for particles that have escaped through the barrier to the outside where classical motion is again allowed (see Refs. [23,24]). With the help of "the imaginary time technique" we shall study in detail the dependence of the pair creation probability on the frequency and the amplitude of the wave.

In addition to the pairs created by the electromagnetic wave via the Schwinger mechanism there is electron-positron pair creation due to the trident process [25] and bremsstrahlung photons. We shall show that the electron-positron pair produced via the Schwinger mechanism can be distinguished from the one appeared due to other mechanisms by the momentum filter, i.e., the momentum of the "Schwinger" pair in the laboratory frame is larger.

The paper is organized as follows. In Sec. II we discuss the properties of the relativistically strong electromagnetic wave in plasma. The probability of pair creation by such a wave is calculated in Sec. III. In Sec. IV the discussions of main results and conclusions are presented.

II. RELATIVISTICALLY STRONG ELECTROMAGNETIC WAVE IN PLASMA

In this section we recover well-known properties of a relativistically strong electromagnetic wave in an underdense collisionless plasma needed for further formulation of the time-varying electric field configuration. A discussion of the wave behavior is based on the results obtained by Akhiezer and Polovin in Ref. [26]. We consider the circularly polarized electromagnetic wave, which is described by the transverse component of the vector potential:

$$\mathbf{A}_\perp = A_0[\mathbf{e}_y\sin(\omega t - kx) - \mathbf{e}_z g\cos(\omega t - kx)]. \quad (3)$$

The wave propagates in the direction of the x axis, its phase velocity equals ω/k, g represents right-handed ($g=1$) or left-handed ($g=-1$) circular polarization, and $A_0 = cE_0/\omega$ with E_0 being the amplitude of the electric field. The dependence of the wave frequency ω on the x component of the wave vector k is given by the dispersion equation

$$\omega^2 = k^2c^2 + \sum_\alpha \frac{\omega_{p\alpha}^2}{[1+(Z_\alpha eA_0/m_\alpha c^2)^2]^{1/2}}, \quad (4)$$

where $\alpha = e,p,i,\ldots$ denotes species in the plasma with the electric charge $Z_\alpha e$, mass m_α, and density n_α, $\omega_{p\alpha} = (4\pi n_\alpha Z_\alpha^2 e^2/m_\alpha)^{1/2}$, and c is a speed of light in a vacuum. By virtue of the plasma electric neutrality condition we have $\Sigma_\alpha Z_\alpha n_\alpha = 0$. Introducing notation for the frequency ω,

$$\Omega^2 = \sum_\alpha \frac{\omega_{p\alpha}^2}{[1+(Z_\alpha eA_0/m_\alpha c^2)^2]^{1/2}}, \quad (5)$$

we rewrite the dispersion equation in the form $\omega = (k^2c^2 + \Omega^2)^{1/2}$.

The electric and the magnetic field are equal to

$$\mathbf{E} = \frac{1}{c}\partial_t\mathbf{A}_\perp = \frac{\omega}{c}A_0[\mathbf{e}_y\cos(\omega t - kx) + \mathbf{e}_z\sin(\omega t - kx)] \quad (6)$$

and

$$\mathbf{B} = \nabla\times\mathbf{A} = kA_0[\mathbf{e}_y\sin(\omega t - kx) + \mathbf{e}_z\cos(\omega t - kx)], \quad (7)$$

respectively.

We see that in a plasma the first invariant of the electromagnetic field J_1 is not equal to zero:

$$J_1 = \frac{1}{2}(B^2 - E^2) = -\frac{\Omega^2}{2c^2}A_0^2 \equiv -\left(\frac{\Omega}{\omega}\right)^2 E_0^2. \quad (8)$$

It vanishes when the plasma density tends to zero, i.e., in a vacuum (or in the limit $A_0\to\infty$). It also vanishes when the charges of plasma species go to zero, because $\Omega \sim \omega_{p\alpha}$, which in its turn $\omega_{p\alpha} \sim Z_\alpha$.

The nonlinear electromagnetic wave in plasmas is also characterized by the dependence of its phase and group velocity on the plasma parameters and on the wave amplitude. From Eq. (4) we find that the phase velocity $v_{ph} = \omega/k$ and the group velocity $v_g = \partial\omega/\partial k$ are related to each other via expression $v_{ph}v_g = c^2$.

Now we perform the Lorentz transformation to the reference frame moving with the group velocity along the direction of the wave propagation. The wave frequency and the x component of the wave vector change according to

$$\omega' = \frac{\omega - kv_g}{(1-v_g^2/c^2)^{1/2}} \quad \text{and} \quad k' = \frac{k - \omega v_g/c^2}{(1-v_g^2/c^2)^{1/2}}. \quad (9)$$

Using relationship $v_g = c^2/v_{ph} = kc^2/\omega$, we obtain that in the moving reference frame the wave frequency equals $\omega' = \Omega$ and its wave number vanishes: $k' = 0$. Using the Lorentz invariance of the transverse component of the vector potential we find that in the moving reference frame the magnetic field in the circularly polarized electromagnetic wave vanishes and the electric field, rotating with the frequency Ω, is given by expression

$$\mathbf{E} = \frac{\Omega}{c}A_0(\mathbf{e}_y\cos\Omega t' + g\mathbf{e}_z\sin\Omega t'). \quad (10)$$

Further we shall use a notation $E = (\Omega A_0/c) \equiv (\Omega/\omega)E_0$. See also Ref. [27], where the model case of linearly polarized wave has been considered. The finite amplitude linearly polarized wave propagating in plasmas has both transversal and longitudinal components, oscillating with different frequencies. That is why the problem of the pair production by

this wave is technically more complicated, than the case of circularly polarized wave. It will be considered in our forthcoming publications.

III. PAIR PRODUCTION BY CIRCULARLY POLARIZED WAVE

We consider the problem of pair production in the circularly polarized electric field, using the approach formulated in Ref. [8]. The wave function of the electron in the electromagnetic field has the form

$$\psi_f(\mathbf{p},t) = \int d^3r \, d^3r_0 e^{-ipr} G(r,t;r_0,0) \psi_i(r_0,0). \quad (11)$$

The Green function, according to Feynman, in the coordinate representation has the following asymptotic behavior:

$$G(r,t;r_0,0) \sim (2\pi i t)^{3/2} \exp[iS(r,t;r_0,0)], \quad (12)$$

where S is the action for the electron. For the motion in the homogeneous field $\psi_i \sim \exp(i\mathbf{p}_0\mathbf{r}_0)$. Integration of Eq. (11) with the use of saddle point method yields the following equations for saddle point coordinates (see Ref. [8]):

$$\frac{dS}{dr} - p = 0, \quad \frac{dS}{dr_0} + p_0 = 0. \quad (13)$$

As it is known in the classical mechanics these equations determine the extremal trajectory, which satisfies the Lagrange equations. Therefore with the accuracy up to the preexponential factor we get

$$\psi_f \sim \exp(i\hat{S}) \quad (14)$$

with

$$\hat{S} = \int_0^t \mathcal{L}dt - \mathbf{p}\cdot\mathbf{r} + \mathbf{p}_0\cdot\mathbf{r}_0, \quad (15)$$

where $\hat{S}$ is the reduced action [8], $\mathbf{r}$ is the coordinate of the particle, and $\mathbf{r}_0$ is the initial coordinate and the Lagrangian (see Ref. [4]) is given by expression

$$\mathcal{L} = -m_e c^2 (1 - |\mathbf{v}|^2/c^2)^{1/2} + e\mathbf{A}\cdot\mathbf{v} - e\varphi, \quad (16)$$

which depends on the vector $\mathbf{A}$ and the scalar φ potential of the electromagnetic field and on the charged particle velocity $\mathbf{v} = c\mathbf{p}/(m_e^2 c^2 + |\mathbf{p}|^2)^{1/2}$. Here $|\mathbf{p}|^2 = p_x^2 + p_y^2 + p_z^2$. In particular case, when the charged particle interacts with the electromagnetic wave, the integrand in the action functional (15) can be expressed via the particle energy $\mathcal{E} = (m_e^2 c^4 + |\mathbf{p}|^2 c^2)^{1/2}$ as

$$\hat{S} = -\int_0^t \mathcal{E}dt. \quad (17)$$

An expression for the action remains unchanged in a plasma, if we assume that $\mathbf{A}$ and φ are the vector and scalar potentials of net electromagnetic field acting on the electron or positron. The representation of the electromagnetic wave

field in the form (10) with the frequency Ω given by Eq. (5) implies the use of the macroscopic description of the wave. The analysis of the microscopic structure of the electron-positron pair production in plasmas is beyond the scope of the present publication. However, since similar technique of the Lorentz transformation into the reference frame, moving with the wave group velocity, can be used to simplify the analysis of electron-positron pair production inside the hollow waveguide and in the focus region [30], we think that the main results obtained below in the long wave approximation $[(2\pi c/\omega)^3 n \gg 1]$ will not change drastically.

The probability of pair creation depends on the imaginary part of the action as

$$dw_p \sim \exp\{-2 \text{ Im}[\hat{S}(p)]\} d^3p. \quad (18)$$

To calculate the pair creation probability we need to find the extremal trajectory for the sub-barrier motion of the particle, which minimizes the action. The action functional variation is

$$\delta\hat{S} = -\int_0^t dt \left(\frac{\partial\mathcal{E}}{\partial p_i} \delta p_i + \frac{1}{2} \frac{\partial^2\mathcal{E}}{\partial p_i \partial p_j} \delta p_i \delta p_j + \cdots \right)$$

$$= a_i \delta p_i + \tfrac{1}{2} b_{ij} \delta p_i \delta p_j \quad (19)$$

with the functions

$$a_i = -\int_0^t dt \frac{p_i}{\mathcal{E}} = -\rho_i; \quad b_{ij} = -\int_0^t \frac{dt}{\mathcal{E}} \left(\delta_{ij} - \frac{p_i p_j}{\mathcal{E}} \right). \quad (20)$$

Here $\boldsymbol{\rho} = \mathbf{r}(t) - \mathbf{r}_0$ is the particle displacement during its sub-barrier motion. The requirement of $\text{Im}[\hat{S}]$ being minimal leads to the condition $\text{Im}[\boldsymbol{\rho}] = 0$, which determines the extremal trajectory. We introduce a notation $\mathbf{q}_0$ for characteristic momentum, which corresponds to the extremal trajectory. In the case when the quasiclassical approximation is valid, the momentum spectrum of emerging from under the barrier particles has a sharp peak near $\mathbf{p} = \mathbf{q}_0$ [8]. However the discussion of the exact form of the spectrum of the produced electrons and positrons is beyond the scope of the present paper and it will be carried out elsewhere. The sub-barrier motion is determined by the classical equations:

$$\dot{\mathbf{p}} = e\mathbf{E}, \quad (21)$$

$$\dot{\mathbf{r}} = c \frac{\mathbf{p}}{(m_e^2 c^2 + |\mathbf{p}|^2)^{1/2}}. \quad (22)$$

We should add to these equations of sub-barrier motion the requirement of trajectory to be the extremal one, i.e., $\text{Im}[\boldsymbol{\rho}] = 0$, then for the circularly polarized electric field of the form

$$E_x = 0, \quad E_y = E\cos\Omega t, \quad E_z = E\sin\Omega t, \quad (23)$$

we find

$$p_y = q_y + P \sin \Omega t, \qquad p_z = q_z - P \cos \Omega t, \qquad p_x = q_x, \tag{24}$$

$$\mathrm{Im}[\boldsymbol{\rho}] = \frac{2}{\Omega} \mathrm{Re}\left[\int_0^{\tau_0} \frac{\mathbf{p}}{(m_e^2 c^2 + |\mathbf{p}|^2)^{1/2}} d\tau \right] = 0, \tag{25}$$

where $\tau = -i\Omega t$ and $P = eE/\Omega$. From condition $\mathrm{Im}[\boldsymbol{\rho}] = 0$ we obtain $q_y = q_x = 0$. As we see, when the particle emerges from under the barrier ($t=0$) its momentum is perpendicular to the instantaneous direction of the electric field. Here we encounter the drastic difference of the two-dimensional case of the particle motion (it was first pointed out in Ref. [8]), which is realized in the circularly polarized wave, from the one-dimensional motion considered in Refs. [7,13,24]. In the former case the point of particle emerging from under the barrier is the turning point of the trajectory and therefore the particle momentum should be here equal to zero. In the two-dimensional case the component of particle momentum along the instantaneous direction of the electric field is zero. It is due to the fact that under the barrier this component is purely imaginary. The perpendicular component of the particle momentum can be complex. We use a notation $q_z = Ps$ and rewrite the equations for τ_0 and s in the following form, which corresponds to the equations of Ref. [8] with ellipticity equal to 1:

$$\sinh^2 \tau_0 - (s - \cosh \tau_0)^2 = \gamma^2, \tag{26}$$

$$\int_0^{\tau_0} \frac{s - \cosh \tau}{[\gamma^2 + (s - \cosh \tau)^2 - \sinh^2 \tau]^{1/2}} d\tau = 0, \tag{27}$$

where τ_0 is the singular point of $W(t)$ with $W(t)$ being the action for the particle moving along extremal trajectory. τ_0 has a meaning of sub-barrier motion time and $m_e c/P = \gamma$. The parameter γ was introduced in Refs. [5,6], and plays a role of the adiabaticity parameter, as is easily inferred from the function $g(\gamma)$, defined below, which enters the principal exponential factor in the pair production probability. Indeed, as long as $\gamma \ll 1$, i.e., in the high-field, low-frequency limit the result obtained in the framework of the imaginary method agrees with the nonperturbative result of Ref. [1] for a static, spatially uniform field. Note that γ characterizes the dynamics of particle tunneling through a time-varying barrier and is similar to the well-known Keldysh parameter in the theory of the multiphoton ionization of atoms and ions by laser radiation [31]. Due to the fact that in the present paper we do not use Lorentz factor, the notation γ for the adiabaticity parameter should not cause any ambiguity. The sub-barrier motion time τ_0 is determined by condition $\mathcal{E} = (m_e^2 c^4 + |\mathbf{p}|^2 c^2)^{1/2} = 0$, which leads to Eq. (26). Equation (27) follows from a condition $\mathrm{Im}[\rho_z] = 0$. The behavior of the functions s and τ_0 is presented in Figs. 1(a) and 1(b).

Using τ_0 and s from Eqs. (26) and (27) we can calculate the differential probability of the pair creation. It reads (see Refs. [28,29])
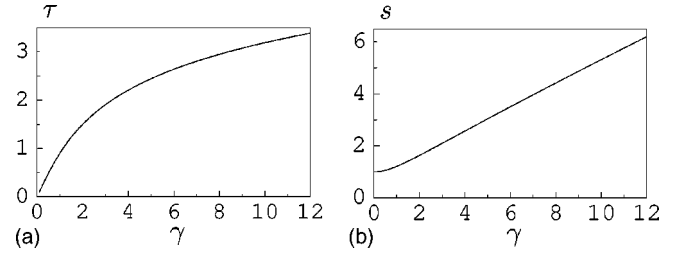

FIG. 1. (a) The dependence of the initial electron (positron) momentum normalized on the amplitude of vector potential s on the parameter γ; (b) the dependence of the electron sub-barrier motion time τ_0 on the parameter γ.

$$dw_p = \exp\left\{ -\frac{\pi}{\varepsilon}\left[g(\gamma) + c_x \frac{p_x^2}{m_e^2} + c_\perp \frac{(q - p_\perp)^2}{m_e^2} \right] \right\} d^3 p, \tag{28}$$

where $\varepsilon = E/E_{cr}$ is the normalized amplitude of the electromagnetic wave in the moving reference frame, $p_\perp$ is the momentum of the pair in the plane perpendicular to the direction of wave propagation. $E_{cr} = 2m_e^2 c^3/e\hbar$ is the Schwinger field, and the functions $g(\gamma)$, $c_x(\gamma)$, $c_y(\gamma)$, and $c_z(\gamma)$ are given by expressions [8]

$$g(\gamma) = \frac{4}{\pi \gamma} \int_0^{\tau_0} K^{1/2} d\tau, \tag{29}$$

$$c_x(\gamma) = g(\gamma) + \frac{\gamma}{2} \frac{dg(\gamma)}{d\gamma}, \tag{30}$$

$$c_\perp(\gamma) = -\gamma \frac{dc_x(\gamma)}{d\gamma}, \tag{31}$$

where $K = 1 - [\sinh^2 \tau - (s - \cosh \tau)^2]/\gamma^2$. According to Ref. [24] the function $g(\gamma)$ demonstrates typical behavior of the pair creation in the linearly polarized electric field. The function $g(\gamma)$ is presented in Fig. 2 along with two typical examples from Ref. [24].
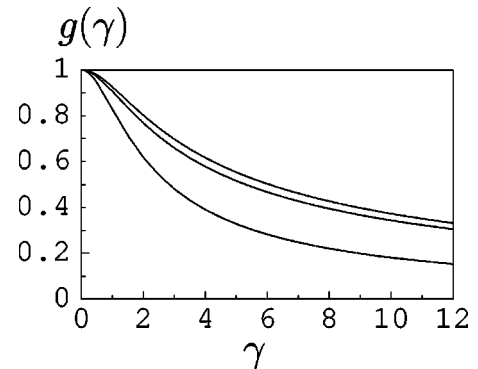

FIG. 2. $g(\gamma)$ vs γ. The upper curve corresponds to the circularly polarized electromagnetic wave, which coincides with the result of Ref. [8], next one corresponds to the linearly polarized electromagnetic wave $\sim \cos \omega t$, the last one also to the linearly polarized wave, but with $1/\cosh \omega t$ time dependence.

The total probability of pair creation can be represented as a sum of probabilities of many-photon processes

$$W = \sum_n w_n, \tag{32}$$

where w_n is dw_p integrated over d^3p with energy conservation in the multiphoton process taken into account (see Refs. [6,7]). In the limit of small adiabaticity parameter ($\gamma \ll 1$), typical for nowadays lasers [24], we get for the total probability of pair creation per unit volume per second

$$W \approx \frac{c}{4\pi^3 \lambda_e^4} \varepsilon^2 \exp\left[-\frac{\pi}{\varepsilon} g(\gamma) \right]. \tag{33}$$

We should note that unlike the case of linear polarization here there is no additional factor $\sqrt{\varepsilon}$ (see Ref. [24] for details) compared to a constant field. It is due to the fact that in the limit $\omega \to 0$ the circularly polarized field becomes constant one.

However the parameter γ is not convenient for studying the probability of pair creation, because the probability depends on ε and γ, which are not independent. These parameters are related as

$$\gamma = \frac{m_e c}{P} = \frac{m_e c \omega}{eE} = \frac{2\pi \lambda_e}{\lambda \varepsilon}, \tag{34}$$

where $\lambda_e = 3.86 \times 10^{-11}$ cm is the electron Compton length. Therefore it is more convenient to study the probability dependence on the electromagnetic wave wavelength λ and on the parameter ε. In addition, the electromagnetic wave is described in terms of amplitude (ε), wavelength (λ), and its shape, which determines function $g(\gamma)$.

In order to understand at what wavelength the shape of the pulse begins to play an important role, we present the dependence of $g(\varepsilon)$ on parameter ε in Fig. 3(a) for different values of λ. It can be seen from this figure that for $\lambda < 10^{-7}$ cm the dependence of $g(\varepsilon)$ on ε should be taken into account. However all the nowadays lasers are far from this limit (see Ref. [24]). This effect can manifest itself only in future x-ray lasers. Moreover there is another problem with detecting this effect. One should have rather small ε [see Fig. 3(a)], at which the pairs are produced at very small rate. As it can be seen from Fig. 3(b), where the dependence of pair creation probability on ε for different values of λ is shown, for $\lambda = 10^{-7}$ cm one pair is created at $\varepsilon \sim 0.08$ in the volume $(10^{-6}$ cm$)^3$ and in 500 fms. To elucidate this point we present Figs. 3(c) and 3(d), where the dependence of g and W on λ for $\varepsilon = 0.08$ are shown, respectively.

We should notice that the initial parameters of the electromagnetic wave in vacuum enter the expression for pair creation probability through parameters γ and ε:

$$\gamma = \frac{m_e c}{P} \frac{\omega}{\Omega} \quad \text{and} \quad \varepsilon = \frac{E}{E_{cr}} = \frac{E_0}{E_{cr}} \frac{\Omega}{\omega}, \tag{35}$$

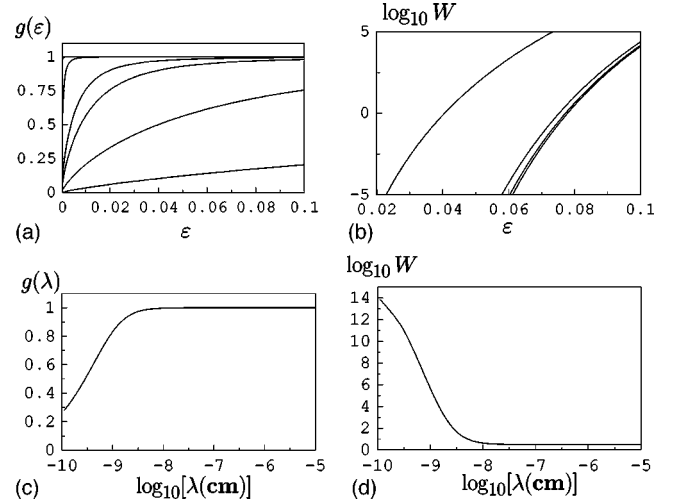

FIG. 3. (a) The dependence of g on ε for different values of λ: 10^{-10} cm, 10^{-9} cm, 5×10^{-9} cm, 10^{-8} cm, 10^{-7} cm, and 10^{-6} cm, from bottom to top; (b) the dependence of $\log_{10}W$ in the volume $(10^{-6}$ cm$)^3$ and in 500 fms on ε for different values of λ: 10^{-9} cm, 5×10^{-9} cm, 10^{-8} cm, and 10^{-7} cm from top to bottom; (c) the dependence of g on λ for $\varepsilon = 0.08$; (d) the dependence of $\log_{10}W$ in the volume $(10^{-6}$ cm$)^3$ and in 500 fms on λ for $\varepsilon = 0.08$.

where E_0 is the amplitude of the electric field in the laboratory reference frame. So the electric field amplitude effectively decreases in plasma and γ increases.

In the reference frame moving with the wave group velocity using expression (10) we find that the electron momentum is equal to

$$\mathbf{p}' = P[\mathbf{e}_y(s - \cos\Omega t') - \mathbf{e}_z \sin\Omega t']. \tag{36}$$

The electron energy is

$$\mathcal{E}' = \left\{ m_e^2 c^4 + 2P^2 \left[1 + \frac{s^2}{2} - s\cos\Omega t' \right] \right\}^{1/2}. \tag{37}$$

Performing the Lorentz transformation to the laboratory frame we obtain the electron energy

$$\mathcal{E} = \frac{\omega}{\Omega} \left[m_e^2 c^4 + 2P^2 \left(1 + \frac{s^2}{2} - s\cos\chi \right) \right]^{1/2} \tag{38}$$

with $\chi = \omega t - kx$ and its longitudinal momentum

$$p_x = \left\{ \left[\left(\frac{\omega}{\Omega} \right)^2 - 1 \right] \left[m_e^2 c^4 + 2P^2 \left(1 + \frac{s^2}{2} - s\cos\chi \right) \right] \right\}^{1/2}. \tag{39}$$

The transverse momentum is given by

$$\mathbf{p} = \frac{P}{c} \{ \mathbf{e}_y[s - \cos\chi] - \mathbf{e}_z \sin\chi \}. \tag{40}$$

In Fig. 4 we present averaged energy and longitudinal momentum versus γ. The averaging was performed on the phase of the pair creation, i.e., by calculating integral
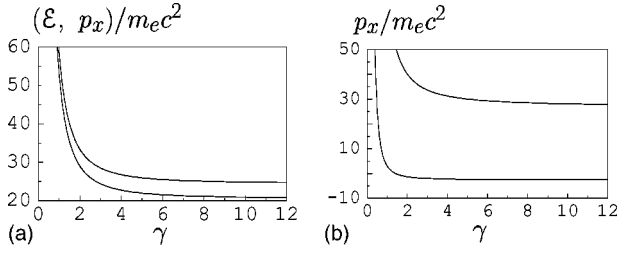
FIG. 4. (a) The energy and the momentum of produced pairs in the laboratory frame, averaged over time, vs parameter γ for $\omega/\omega_p = 5$; (b) the momenta of "Schwinger" pairs (upper curve) and "trident" pairs in the laboratory frame, averaged over time, vs parameter γ.

$$\langle \cdots \rangle = \frac{\omega}{2\Omega\pi} \int_0^{2\pi} (\cdots) d\chi. \tag{41}$$

We see that in the limit of large P and ω/Ω the energy and the longitudinal component of the particle are proportional to $m_e c^2(\omega P/\Omega)$ and $m_e c(\omega P/\Omega)$.

In addition to the pairs created by the electromagnetic wave via the Schwinger mechanism there is electron-positron pair creation due the trident process (see Ref. [25]). The electron (positron) is created by the trident process in the laboratory reference frame with the negligible small transverse momentum and then it gains the transverse momentum equal to

$$\mathbf{p}' = \frac{P}{c}[\mathbf{e}_y(\cos\Omega t' - \cos\Omega t_0') - \mathbf{e}_z(\sin\Omega t' - \sin\Omega t_0')]. \tag{42}$$

Contrary to the particles created via the Schwinger mechanism these electrons (positrons) have in the moving reference frame nonzero longitudinal momentum. It is equal to

$$p'_x = -\frac{m_e v_g}{(1 - v_g^2/c^2)^{1/2}} = -m_e c\left[\left(\frac{\omega}{\Omega}\right)^2 - 1\right]^{1/2}. \tag{43}$$

In Fig. 4(b) we present the dependence of the longitudinal momenta in the laboratory frame of trident electrons (lower curve) and "Schwinger" electrons on parameter γ. It is obvious that these two mechanisms of pair production can easily be distinguished, because the longitudinal momentum of Schwinger electrons is larger. The same analysis can also be applied to the electron-positron pairs produced by the bremsstrahlung photons [32], interacting with plasma particles.

We should note that one of the possible explanations of this phenomenon is that Schwinger pairs are produced on vacuum, while in all other mechanisms the pairs are produced on moving plasma particles.

IV. CONCLUSIONS

In the present paper we considered the problem of the electron-positron pair creation by the circularly polarized laser pulse in a plasma via the Schwinger mechanism. The representation of the electromagnetic wave field in the form (10) with the frequency Ω given by Eq. (5) corresponds to the use of the macroscopic description of the electromagnetic wave. The analysis of the microscopic structure of the electron-positron pair production in plasmas is beyond the scope of the present publication. However, since similar technique of the Lorentz transformation into the reference frame, moving with the wave group velocity, can be used to simplify the analysis of electron-positron pair production inside the hollow waveguide and in the focus region [30], we think that the main results obtained above in the long wave approximation $[(2\pi c/\omega)^3 n \gg 1]$ will not change substantially. On the other hand the elaboration of the microscopic description will particularly determine the applicability of the approximation used.

Using the properties of the dispersion equation for the electromagnetic wave in plasma, we carried out the calculation of the probability in the reference frame moving with the group velocity of the wave. The great simplification of the problem has been achieved owing to the fact that in this reference frame the magnetic field of the wave vanishes and the problem has been reduced to the problem of the pair creation in rotating electric field.

In the present paper we used "the imaginary time method" to calculate the probability of pair creation by the circularly polarized electric field. There arises a problem of validity of the quasiclassical tunneling time and the imaginary time method. The calculations of the pair production by the time-varying electric field carried out within the frameworks of the imaginary time method [6] and Dirac theory [9] agree to each other up to the preexponential factor accuracy. It is also well known that the imaginary time technique has been widely used in the problem of the atom ionization by strong electromagnetic wave. Theoretical and experimental results relevant to the ionization problem can be found in Refs. [33,34] and in cited literature therein. See also Ref. [35].

We noticed that there is a drastic difference between the one-dimensional and the two-dimensional cases of particle motion below the energy barrier, i.e., in the one-dimensional case the particle emerges from under the barrier with zero momentum. In two-dimensional case the particle is produced with nonzero momentum perpendicular to the instantaneous direction of the electric field, which was first pointed out in Ref. [8].

In the moving reference frame we calculated the probability of the pair creation. We found that one pair is produced in $(10^{-6}\text{ cm})^3$ volume in 500 fms when the field amplitude reaches a value about 0.08 of the critical QED field for lasers with the wavelength $\lambda = 10^{-4} - 10^{-8}$ cm. This value of the field amplitude in plasma in the moving frame corresponds to initial field amplitude in vacuum $E_0 = E\omega/\Omega$. We also studied the dependence of pair creation probability on the wavelength of the electromagnetic wave.

We notice that the electron-positron pairs created via the Schwinger mechanism can be distinguished from the pairs created in the trident reaction during interaction of the plasma electrons with nuclei as well as from the pairs produced by bremsstrahlung photons. The electron-positron pairs created via the Schwinger mechanism in the laboratory reference frame have the longitudinal momentum larger than the pairs appeared due to the trident reaction and bremsstrahlung induced pair production.

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